

Irrigation and Drainage Engineering

(Soil Water Regime Management)

(ENV-549, A.Y. 2025-26)

4ETCS, Master option

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Platform of Hydraulic Constructions



Lecture 3-1. Surface irrigation: hydraulic design of canals

Review of free-surface flow and hydraulic design of canals



Flow classification

Type of flow

- **permanent** $\frac{\partial}{\partial t} = 0$
- **transient** $\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = 0$

$\left\{ \begin{array}{l} \text{- uniform} \\ \text{- varied} \end{array} \right. \quad \frac{\partial}{\partial x} = 0$

$\left\{ \begin{array}{l} \text{- gradually} \\ \text{- suddenly} \end{array} \right.$

x : longitudinal coordinate
 Q : flow discharge
 A : flow area

Flow regime (Reynolds number Re)

- **laminar** $Re < 500$
- **turbulent** $Re > 2000$

$$Re = \frac{U R}{\nu}$$

U : mean flow velocity

R : hydraulic radius

ν : kinematic viscosity = ν (Temperature)

$$\nu \propto 10^{-6} \text{ m}^2/\text{s}$$

Flow kinematics (Froude number Fr^1)

- **fluvial** $Fr < 1$ ($y > y_c$)
- **critical** $Fr = 1$ ($y = y_c$)
- **torrential²** $Fr > 1$ ($y < y_c$)

$$Fr = \frac{U}{\sqrt{g y}}$$

y : water depth

y_c : critical water depth



¹ Compare the kinetic energy and the potential energy of the water flow

² Waves can't propagate upstream

Flow regime – Reynolds number

Characterises the relative importance of the forces of inertia and viscosity:

$$Re = UL / \nu$$

(U : mean flow velocity; L : characteristic dimension,
 ν : kinematic viscosity of water)

Channel flow :

$$Re = UR / \nu$$

(R : hydraulic radius)

- Laminar flow regime: viscosity forces predominate over inertia forces ($Re < 500$)
- Turbulent flow regime: the forces of viscosity are of little importance compared with the forces of inertia ($Re > 500$)¹. Velocities fluctuate randomly around mean values. This is the type of flow generally encountered.

1 Some authors introduce a transition regime for $500 < Re < 2000$ and therefore place the lower limit of the turbulent regime at 2000.

For pressure flow in circular pipes of diameter D : $R = D/4$, so that the limit between laminar and turbulent flow is no longer 500, but 2000.

Uniform free-surface flow

Total hydraulic energy per unit weight of water: $H = z + \frac{p}{\rho_w g} + \frac{U^2}{2g} = z + y + \frac{U^2}{2g}$

Flow conditions = f {

- flowrate
- channel characteristics (slope, roughness, geometry, etc.)

$\frac{dH}{dx} = -S$ Loss of energy, positive downstream

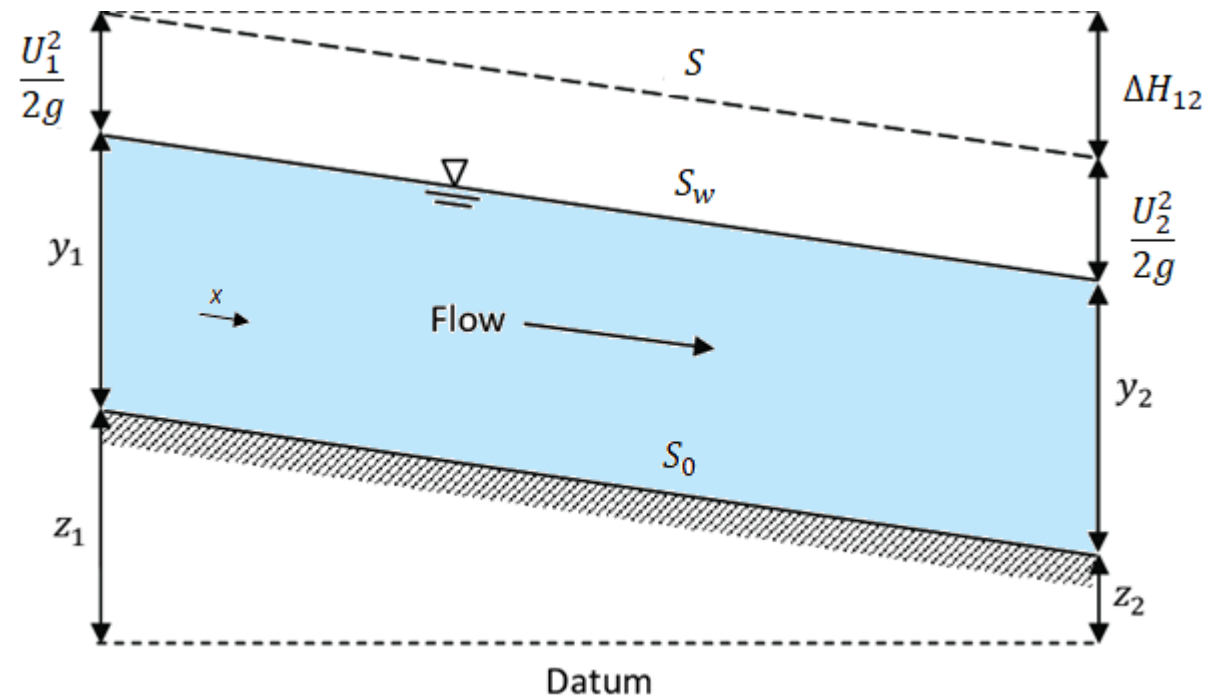
$\frac{dz}{dx} = -S_0$ Bed slope, positive downstream

$\frac{dy}{dx} = -S_w$ Surface slope, positive downstream

$\frac{\partial}{\partial x} = 0$ All the variables are constant along the longitudinal coordinate, x

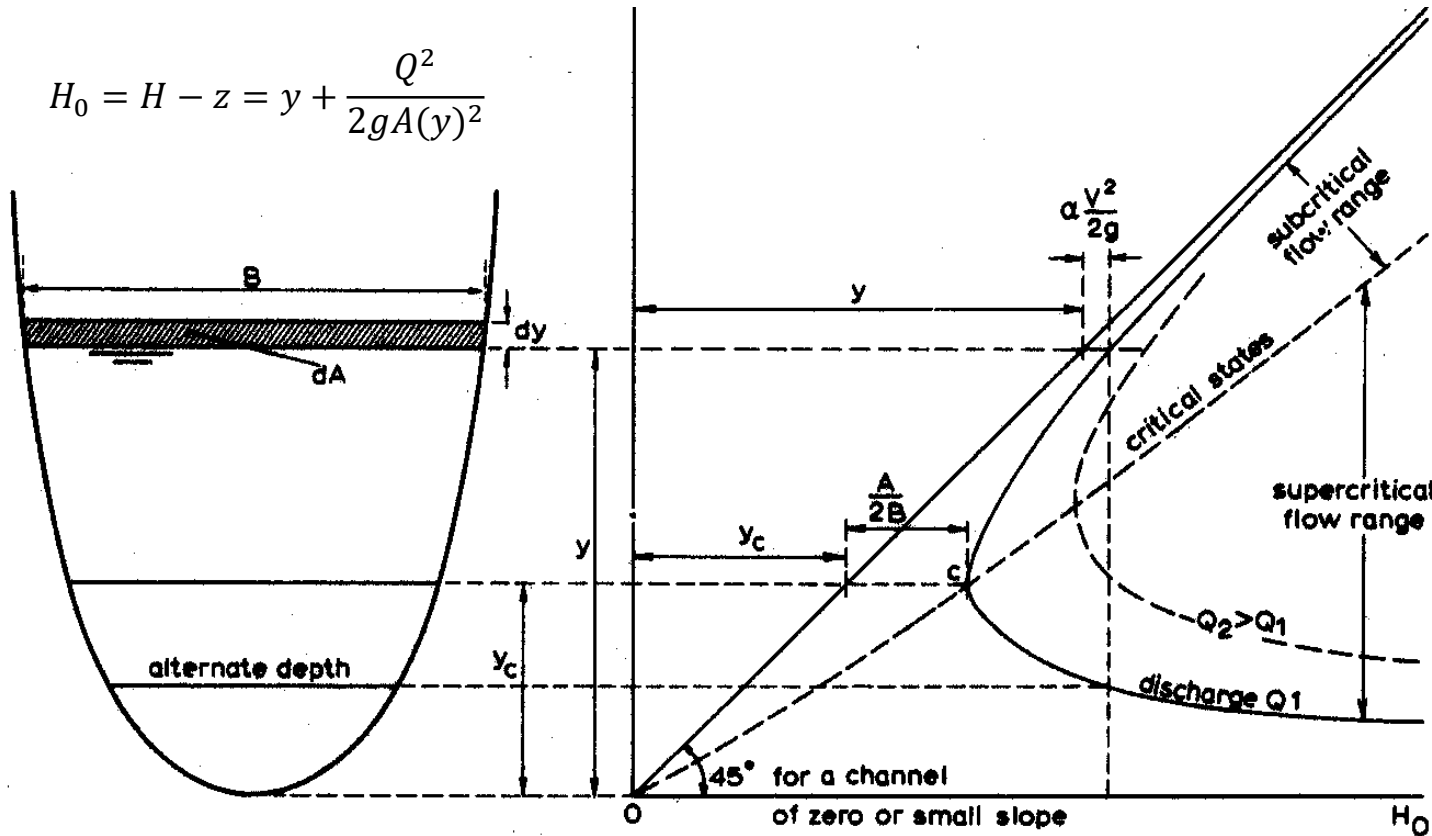
e.g., $\frac{\partial S}{\partial x} = 0$

$$S = S_0 = S_w$$



Specific energy

H_0 : average energy in a section of channel, relative to the bottom of the channel



For a given channel, a given flow and a given specific energy, there are 2 conjugate depths, one corresponding to the torrential regime, the other to the fluvial regime.

- $y > y_c$: **fluvial** regime
- $y < y_c$: **torrential** regime
- $y = y_c$: **critical** regime

$$y_c = y \left(\frac{\partial H_0}{\partial y} = 0 \right)$$

At point C, the specific energy H_0 is minimal.

Point C is the critical point to which the critical height y_c corresponds.

For rectangular cross section $A(y) = B y$

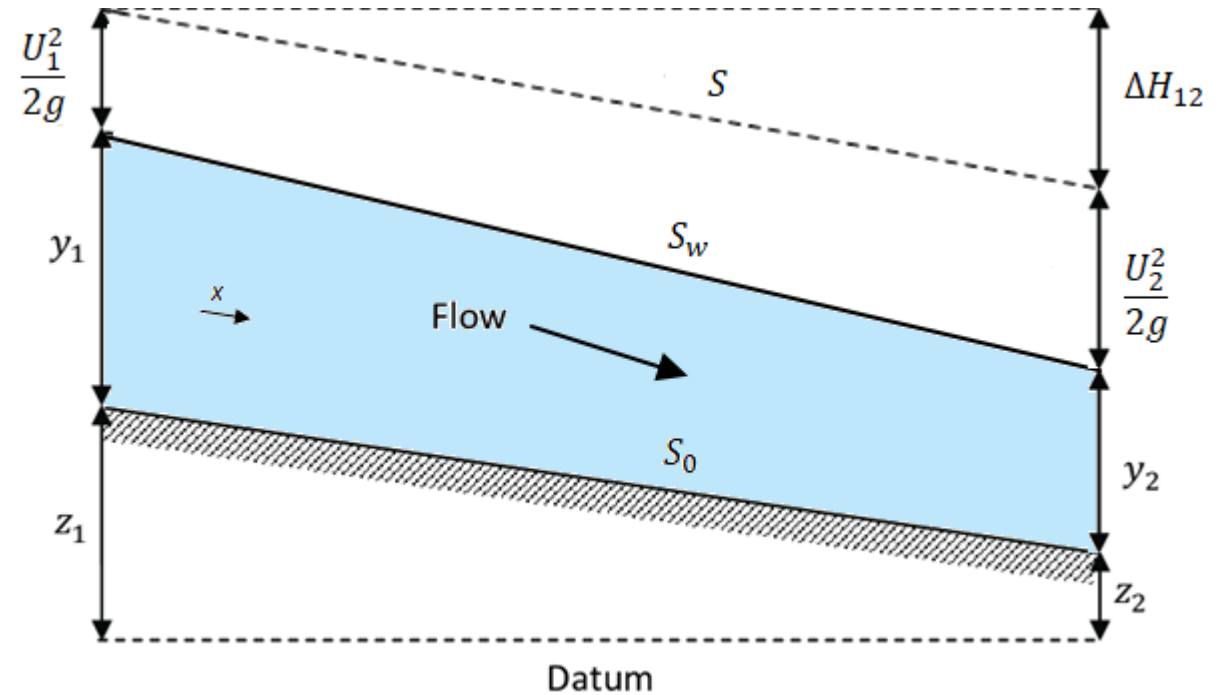
$$y_c = \sqrt[3]{\frac{Q^2}{g B^2}}$$

Permanent free-surface flow

- Flow conditions = f {
- flowrate
 - channel characteristics (slope, roughness, geometry, etc.)
 - presence of point structures and diversions

$\frac{dQ}{dx} = 0$ Flow discharge is constant

$S \neq S_0 \neq S_w$



Water surface profiles in permanent regime

Total hydraulic energy per unit weight of water: $H = z + y + \frac{Q^2}{2g A^2}$

$$\frac{dH}{dx} = \frac{dz}{dx} + \frac{dy}{dx} + \frac{Q^2}{2g} \frac{d}{dx} \frac{1}{A^2} \quad \Rightarrow \quad S_0 - S = \frac{dy}{dx} - \frac{Q^2}{g A^3} \frac{dA}{dx}$$

By chain rule

$$S_0 - S = \frac{dy}{dx} - \frac{Q^2}{g A^3} \frac{dA}{dy} \frac{dy}{dx} \quad \Rightarrow \quad S_0 - S = \frac{dy}{dx} - \frac{Q^2 B}{g A^3} \frac{dy}{dx}$$

$$\frac{dA}{dy} = B \quad \text{Channel width}$$

$$\sqrt{\frac{Q^2 B}{g A^3}} = Fr \quad \text{Froude number}$$

$$\frac{dy}{dx} = \frac{S_0 - S}{1 - Fr^2}$$

We can make a qualitative analysis based on the sign of numerator N, and denominator D

$$\begin{aligned} \text{e.g., } S_0 > S &\rightarrow N > 0 \\ Fr > 1 &\rightarrow D < 0 \end{aligned} \quad \Rightarrow \quad \frac{dy}{dx} < 0$$

Water surface profiles in permanent regime

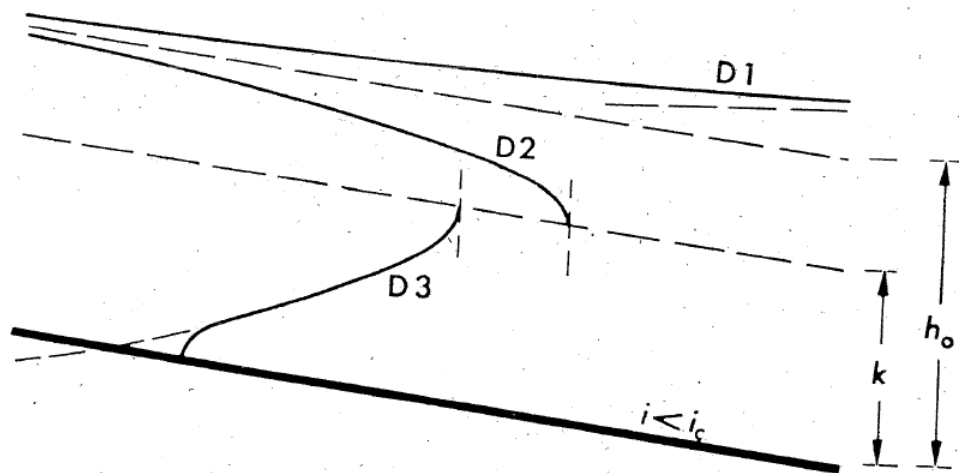


FIG. 10.21

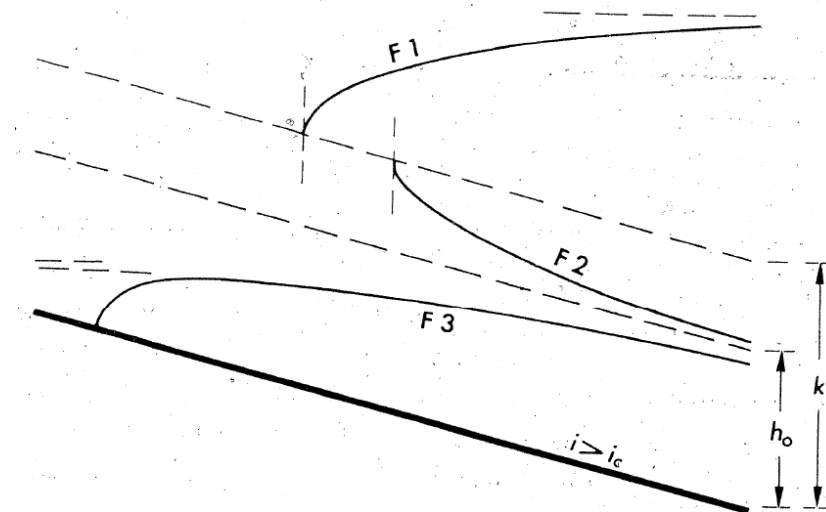


FIG. 10.22

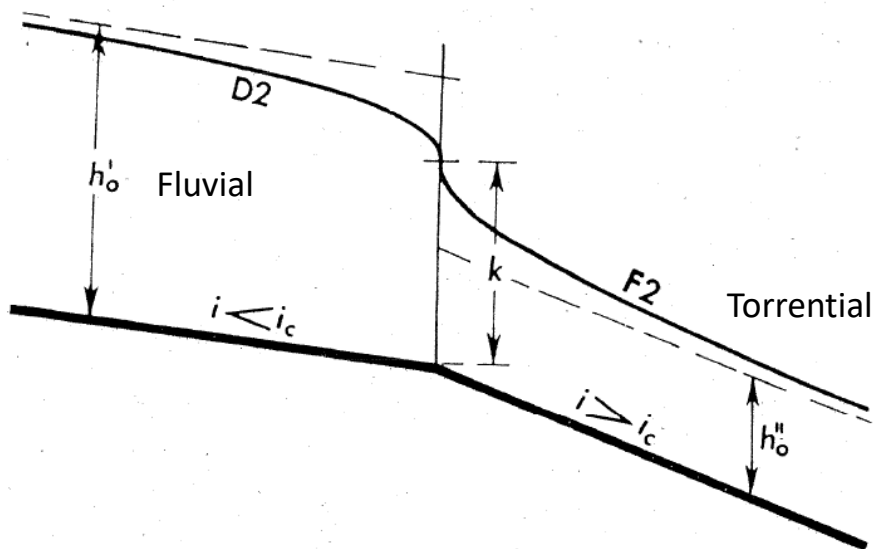


FIG. 10.26

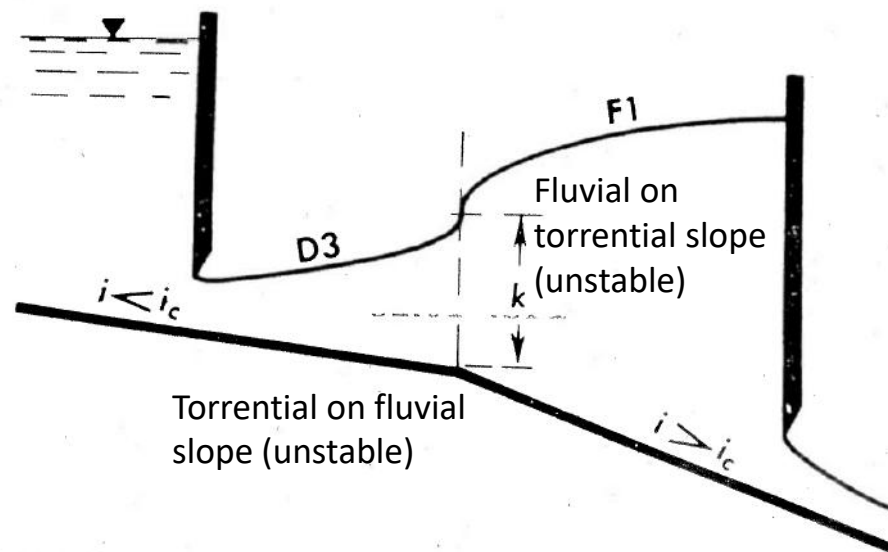
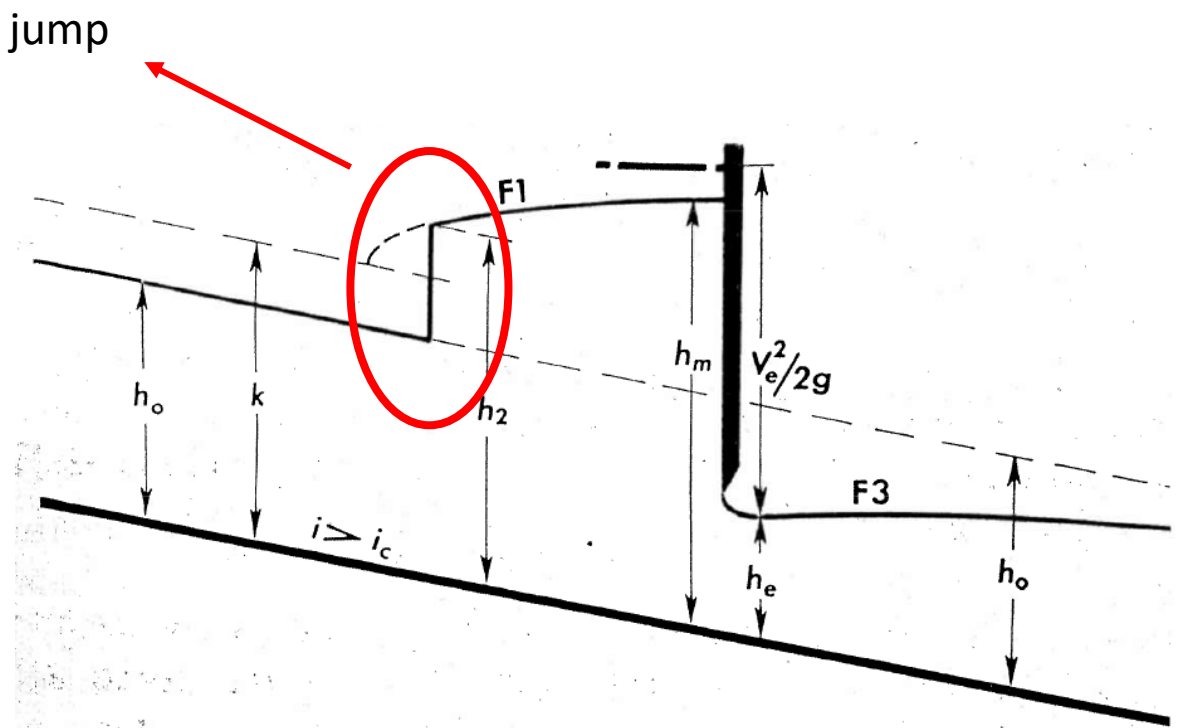
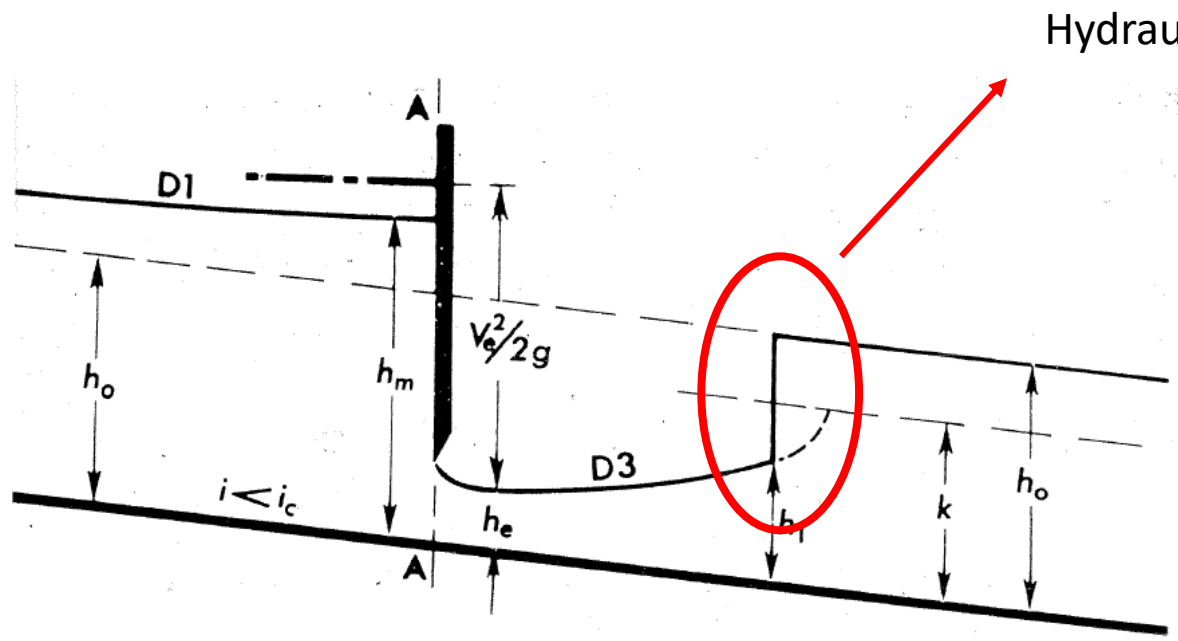


FIG. 10.27

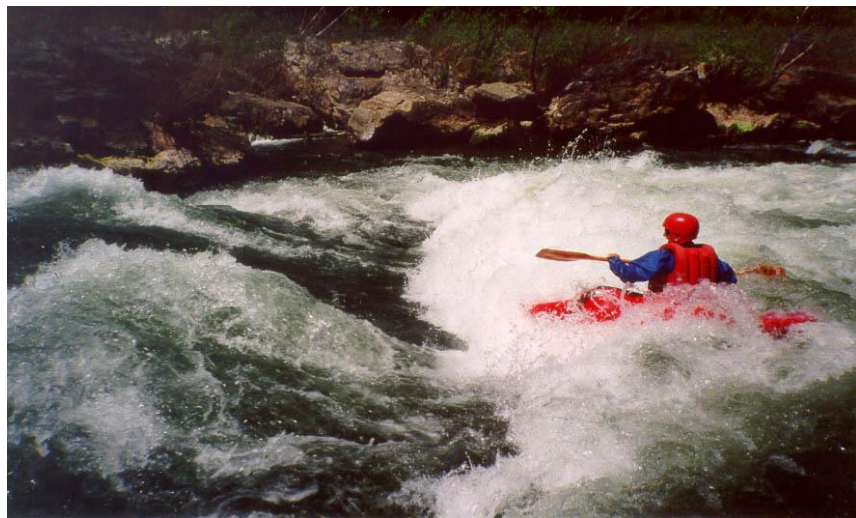
Passage below a sluice gate



Hydraulic jump

Abrupt transition from upstream supercritical flow to downstream subcritical flow

In natural rivers



Downstream of weirs



At home



Examples of hydraulic jumps

<https://www.youtube.com/watch?v=7tjf8HWiR3Y>

Passage below a sluice gate and lateral derivations

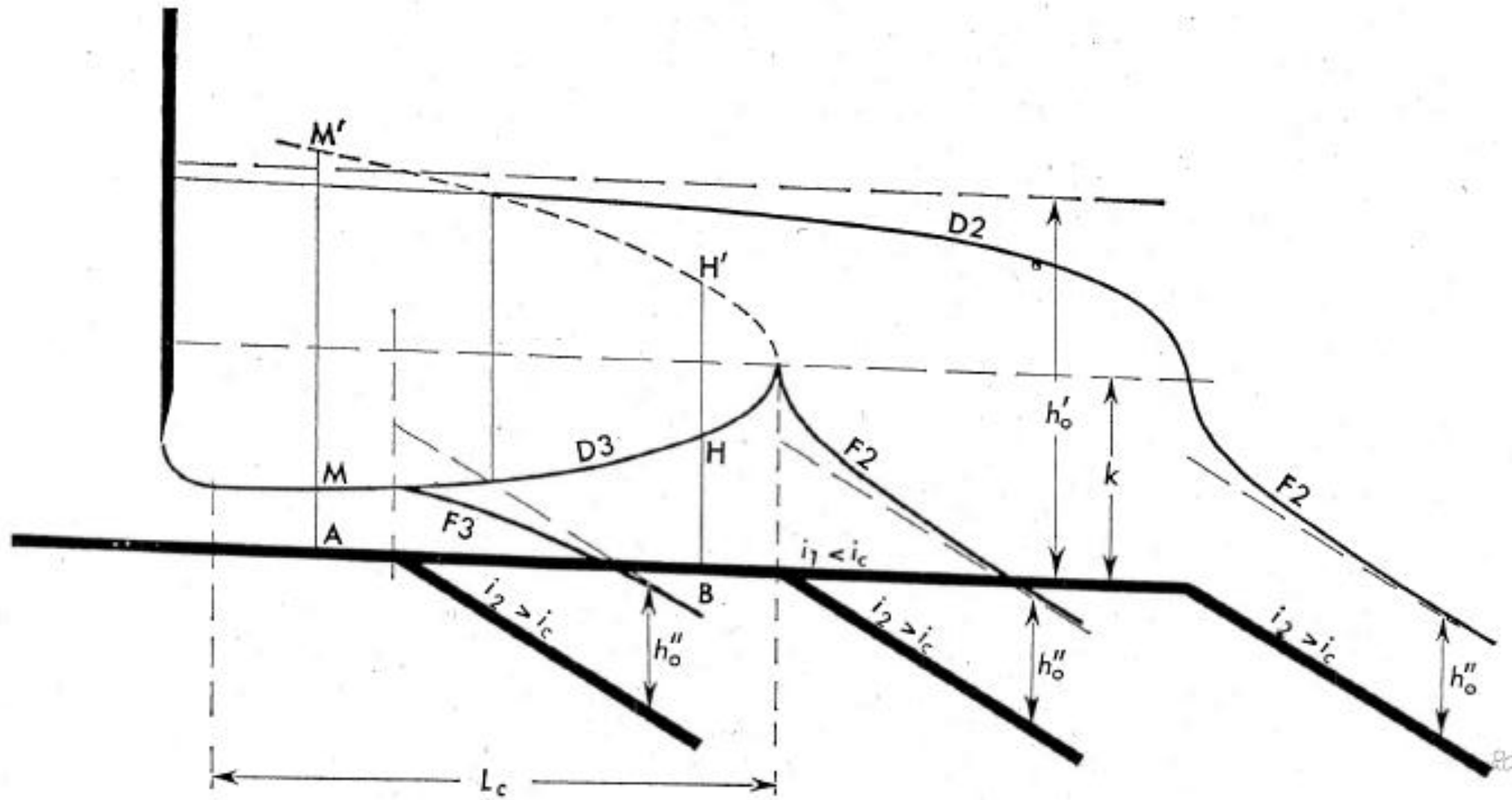


FIG. 10.38

Change of bed roughness – downstream increase

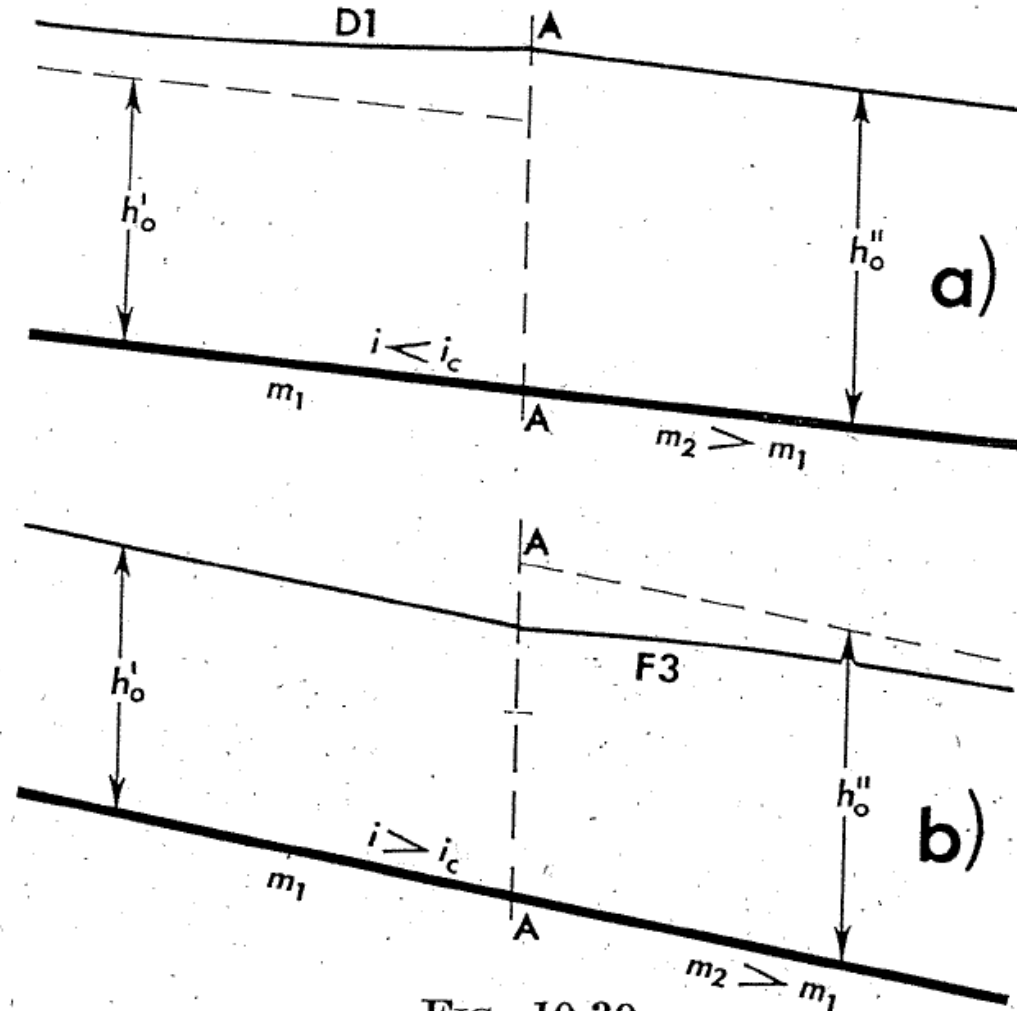


FIG. 10.39

Change of bed roughness – downstream decrease

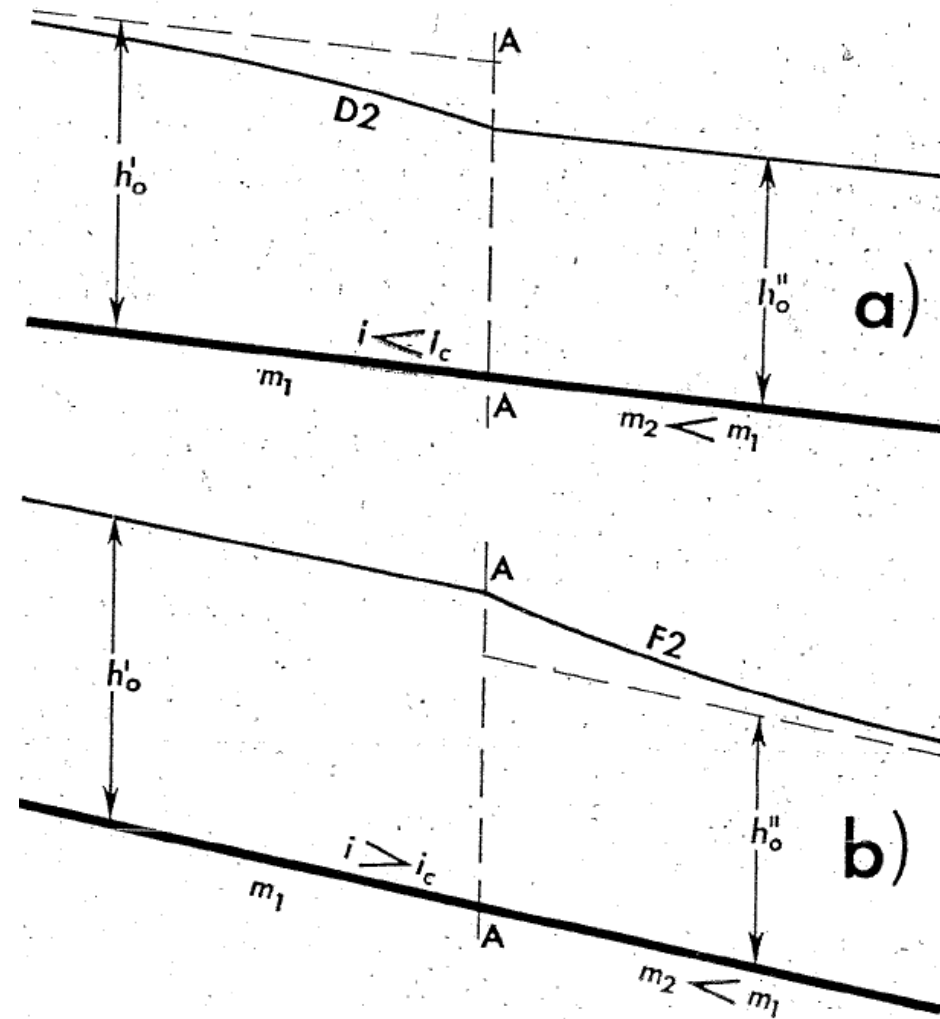
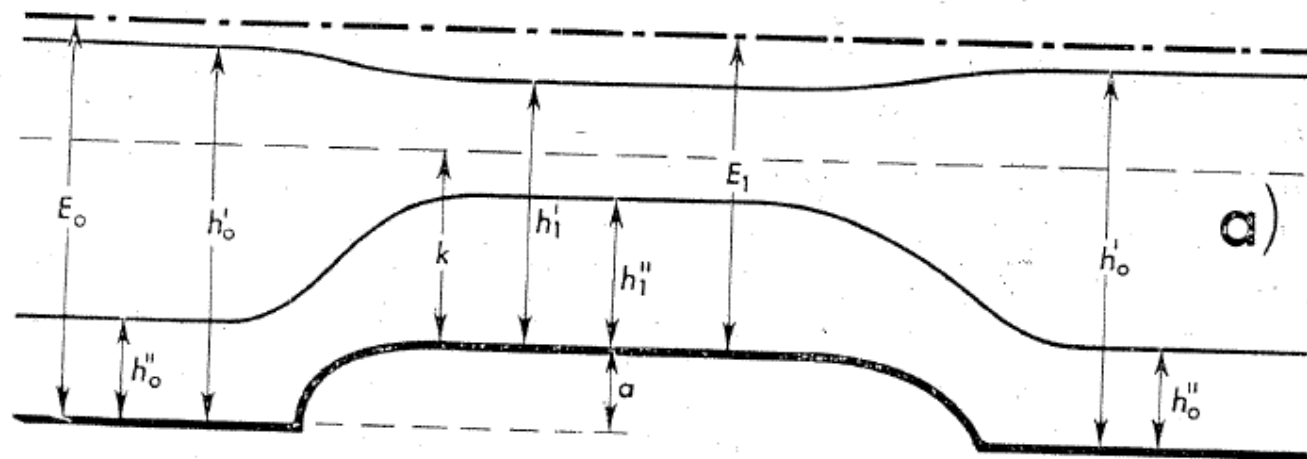


FIG. 10.40.

Passage over a low obstacle



a : height of the obstacle

Low obstacle

$$H_0 - a > H_{\min}$$

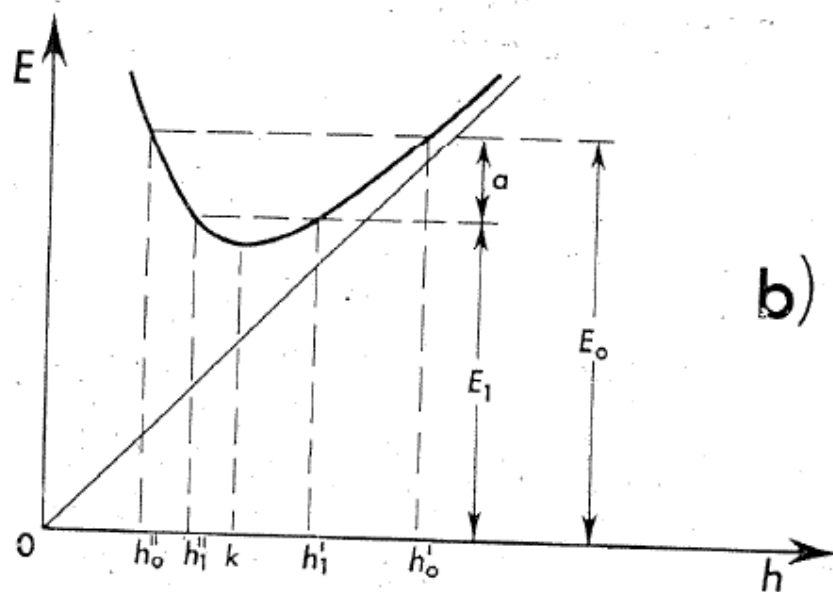
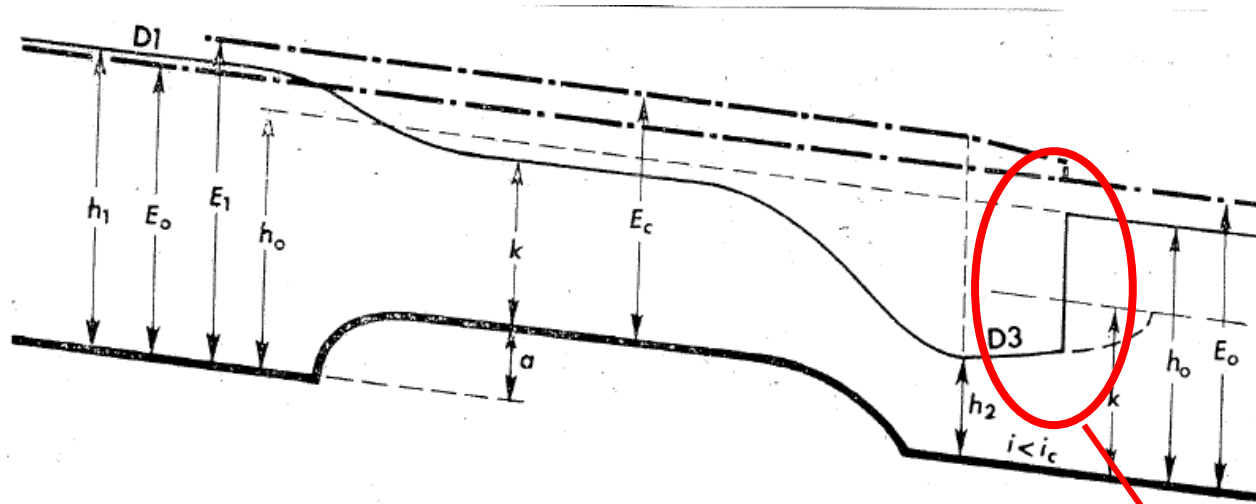


FIG. 10.41

Passage over a high obstacle – fluvial regime



a : height of the obstacle

High obstacle

$$H_0 - a < H_{\min}$$

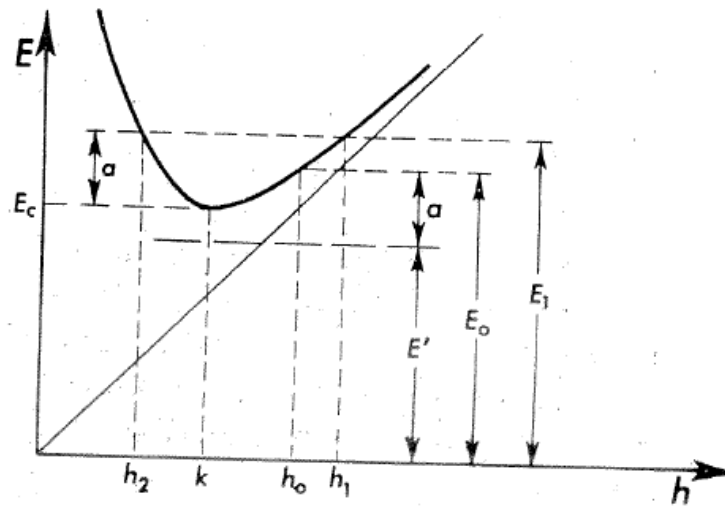


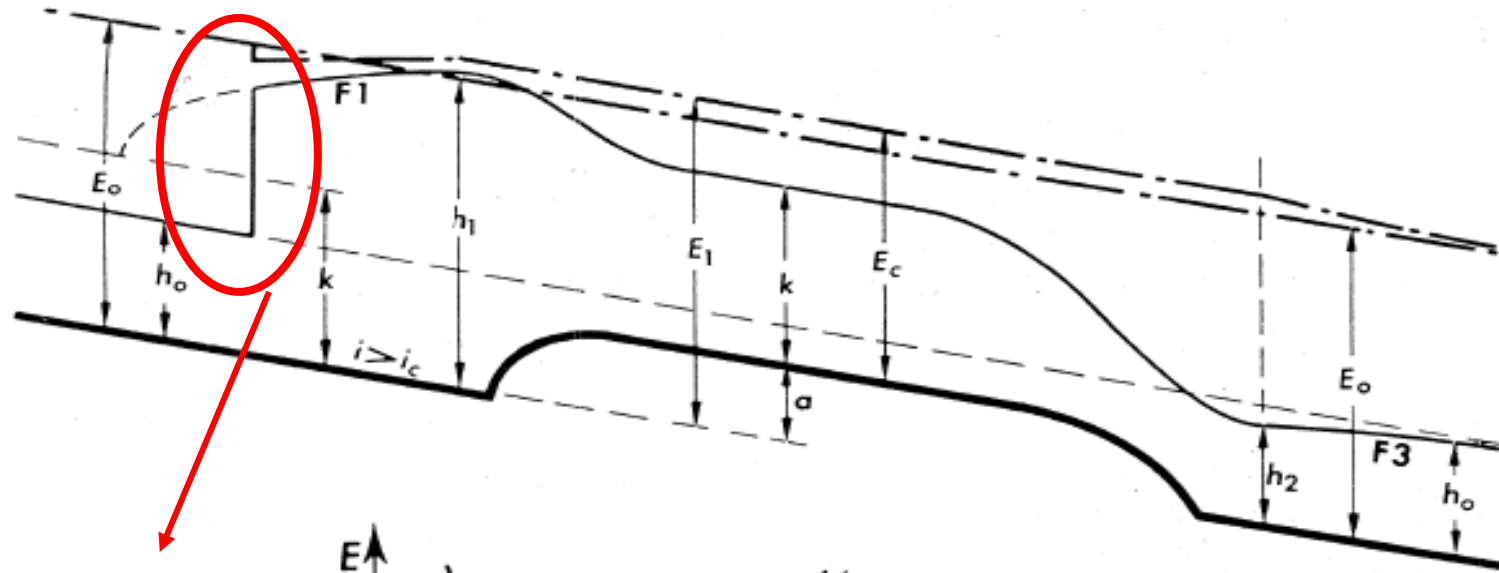
FIG. 10.42

Hydraulic jump

Example in a laboratory flume

<https://youtu.be/xGTntCtQYmA?t=160>

Passage over a high obstacle – torrential regime



a : height of the obstacle

High obstacle

$$H_0 - a < H_{\min}$$

Hydraulic jump

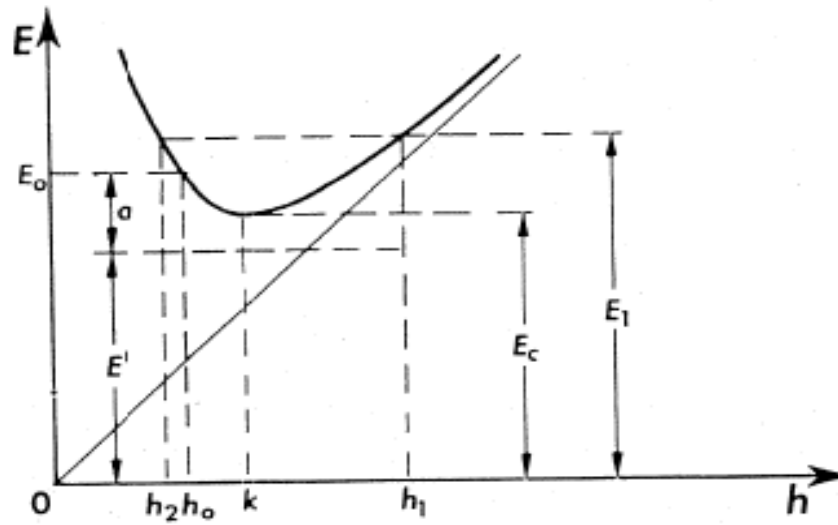


FIG. 10.43

Example in a laboratory flume

<https://youtu.be/KzL4dPGmWss?t=420>